



# 85FOBUBINTDSFFOJOH BOE EJBHOPTUID UFTUIOH OPS OF UBM DI SPN PTPN BINBOE HFOFUID DPOEJUPOT

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- %2##2&;ZR[^Y MUUZ\_T[a)PNQQYY aZUOMOPa\_UZSOOM #\_UY \XQNZPQ[Z\_U\_OZ`XNZSaNSOCTOZ PU\_Oa\_UZSUZbQ\_USMUUZ\_t c UT Q[ZRYY MUUZ`TM\_`TQUZR[^Y MUUZ TM\_NOOZ aZPQ^`[[PNZP c UT `TM aZPO^\_NZPUZS P[OaY OZ`OPž
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Sž FTQaZPQ^\_NZPUZS `TMHURMOT^{Y [\_Y Q[^SQZQUOQ ZPUUZ U\_PUXSZ[\_QPH Mc [Y NZ ONZ OT[[\_Qc TQTQ^`[ Q Z`UZaQ[^`QY UZMQ`TQ\^QSZNZQežI\_TQQMQ ZPUUZ TM\_ NQQZ PUXSZ[\_QPH \MQZ`\_\_T[aXP NQSUbQZ\_aRR000Z` UZR[ Y MUUZ 'QSMPUZS `TQMQU V XSH M\_[ QUMUZ\_HMZP UY \XQMUUZ\_[R`TM PUXSZ[\_U\_Pa^UZS\^QSZNZQeH`TQZQc N[ 'Z \Q'UP NZP NQe[ZPH UZ [ ^PQ^`[ Y MYQUZR[ Y OP PQQUUZ\_NN[ a` `TQJ^[\`UZ\_Ž "### FTQYQ\_T[aXP NQNZ M\_a^NZQQ`TM 'QSMPXQ\_[R`TQJ PQQUUZ L C [ Y QZ c VXNQ[ RRO'CP Q[ aZ\_QXXZS NZP 'QQQUbQ[ZS[ UZS OMQNZP\_a\\[^\*z;Z `TQQMQ[ RQ Z`UZaVZS `TQ\^QSZNZQeH c [ Y QZ NZP `TQJ^M\*ZQ\_\_T[ aXP NQ\^[ bUPQP c UT NI\^[ \ 'UVQNZ: QZMINXOMQc UT UZPUSUPaNXUQP \ 'Q\MMVUZ\_R[ ^NU\*T NZP ZQ ZMNXY NZNSQY QZ` žFTQ[\`UZ [ RZQ[ ZMNX \NXXXVUbQQMQ\_T[ aXP NQPUQa\_QP R[ ^Q ZPUUZ\_c TQQ`TQ\^[ SZ[\_U U\_\][ ^NZP \NXXVUUZ U M'QXU\_UQ[\`UZ;R`TQe OT[[\_Q`QY UZMUZH`TQE ZQQP`[ VZ[c `TMI`TQY [ PQ[ R`QY UZMUZ Y N& NQUZR&QZQP Ne SQ\_MUZXXXQC CUT E`MQXK6\_Z

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50% RCQ 6@3 / OF6@3 fiNM (P\_OQQZ / Z SI Q Y Y [Z & ORO ^OP `[M\_Z [Z Ž Z b M\_UbQ ^ OZ / M/X Q ` / Z S / @, BFfi a\_Q 6@3\_Q] a OZ OZ S [ ^ M/M & NM (P ` QOTZ [X] Se` [ PQ QO MZ Qa \X] UPe VZ \XMODZ ` MX U\_aQ\_Ne Y Q Ma ^ UZ S OF6@3 VZ ` TQ Y MO Z MX \XM Y MIFTU ` Q ` U T VST Xe\_QZ \_ UUbQ MIP T VST Xe \_ QOURD R ^ ` U [Y e \$# Na` P[Q\_Z[` TMbQ\_a RROUZ ` PUMSZ [ \_ UD MODa ^ Moe `[ ^ Q \XMOD VZ b M UbQ ` Q ` VZ S / AZ R X Q \[ \_ UUbQ NZ P R X Q Z COS MUbQ\_\_ ` VX [ Q a ^ fz; ` c M\_ VZ UU X Xe b MX PM (P NZ P OXZ VD X Xe V Y \XQ Y Q ` Q P M\_ MZ o M Pb NZ OQ P [ ^ QQ Z P Me\_O Q Q Z VZ S ` Q ` R ^ c [ Y Q Z M VZ O Q M Q P XMOX T [[ P [ RT Mb VZ S MOT VP c UT NZ Qa \X] UPe NM (P [ Z Y MO Z MX \& G \ / U ^ M NZ [ ^ Y NX\_O Q Q Z VZ S Q a Xi a X ^ M [ a Z P V ^ Q S a X/ Ue [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q ` U Q / XZ Q [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/ M [ a Z P U ^ Q S a X/ Ue [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q Z [ c Nb NX M XO [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/M [ Z P U ^ Q S a X/ UE [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q Z [ c Nb NX M XO [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/M [ Z P U ^ Q S a X/ UE [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q Z [ c Nb NX M XO [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/M [ Z P U ^ Q S a X/ UE [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q Z [ c Nb NX M XO [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/M [ Z P U ^ Q S a X/ UE [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UPe z 6 M M O Q Z [ c Nb NX M XO [ Z U\_a\_Q VZ ` TQ SQ Z O MX \[ \a X/M [ Z P U ^ Q S a X/ UE [ ^ / \U ] ^ TU ` [ ^ [ R NZ Qa \X] UP Z Q ] U ] / \dot Z Q Z U U [ ^ Q T ` Q D Q U ] A X ( Q ` Q ` Q U ] A X ( Q ` Q ` Q Z Q Z Z Z Z U U D U ) / \dot Z Q Z Z U U D U ] / \dot Z Q Z Q U [ ^ OT [ ^ U Z Q D Q X ] X [ C O ` / OD NZ Z Q [ [ A X/M [ Z \_ Z ^ ] ^ ] ^ ] ^ ] / M \ ^ [ dU' M O X & & ' ff M c [ a XP NQ Q A Q O P R [ Y U \_ a Q U Z X [ C O ` / OD NZ U Z Z Q U ] ^ [ ^ ] ( U Z Q U Q Y Z Q U ] A X ( Z Q ` Q U ] A X ( Z Q ` Q U ] A X ( Q Y Q Z Q U ] A X ( Z Q Y Q Y Q Y Q Y Q Y Q Z Q U ] ^ ] ^ [

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#### 3.5.1 Population-based reproductive carrier screening

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/ PPE Q9BDUDF OPUF	/ SBEF BOE TVCOPSJIOH SFGFSFODFT
$\begin{split} B^{0}(\overline{Z}, \overline{U}, \overline{Z}, \underline{O}, \overline{U}, \overline{Z}, \underline{O}, \overline{U}, \overline{Z}, \underline{U}, \overline{A}, \overline{O}, \overline{U}, $	9[[P\^NOUODZ[`Q_ /Q[Z_OZ_a_ŽNV_/OPfi
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 $\label{eq:local_solution_constraint} RNSUQJ_eZP^{Y} Q_{Q} Q_{D} Q_{D}$ 

[ZQ^000\_UbQPU[^P072<sup>55</sup>

$$\label{eq:linear_cond} \begin{split} & UT `TQUZ` ^[PaOU]Z [RZQ: SQZ[Y ID_Q] a QZQUZS `QQTZU] a Q_{1} QM^{1}Q^{2}QQQZUZS R ^{1} ^MY a XU a PQ [R \\ ^QQQ_ IbQQ[ZPUU]Z_UZ[c NbN0KNNQR ^{1} ^Q[a XQ_c T[TNbQZ[RWTU & TU] ^{e} [RNSQZQ ID PU] ^{PQ2} ^{2\#} FTU \\ _[ZQN%QP cQd NZ POP QM^{1}Q^{2}QQZUZSp_T[a XP [Z & NQ[RQ2 OP UZ `TQQ[Z`Qd` [Rc Q%ZPQRZ OP OKZUDNK \\ ^MTc Ne_R ^{1} ^{O} ^{QZ}NZP ^{[Z}Q_ SQZQ ID Q[ a Z_Q%ZS M_a `[ $& [RNPa X_c UK Q_ `[ UIbQR ^{1} ^M XQM] \\ ~[ZQA COD UIC OP UI (D C C ^{4}) \\ ~[ZQA COD UIC OP UI (D C C ^{4}) \\ ~[ZQA COD UIC OP UI (D C C ^{4}) \\ ~[ZQA COD UIC OP UI (D C C ^{4}) \\ ~[ZQA COD UIC OP UIC OP C ^{4}) \\ ~[ZQA COD UIC OP UIC OP C ^{4}) \\ ~[ZQA COD UIC OP UIC OP C ^{4}) \\ ~[ZQA COD UIC OP C ^{4}) \\ ~[ZQA$$

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 $\label{eq:constraint} \end{tabular} \end{t$ 

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#### 4. Governance

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:  $M_0 a \tilde{Z} [Z_PMQVZ] c XQPSQMN[a^TQQa^QZ^_OQZVZSY [PNXUUQ_N_0MXMVQNZPVZ c TM] _Q^UZ_S_TQe QNZ NQVY NXQY QZ^QPZ$ 

 $\label{eq:linear} 4QMNXQ^{[\[\]}bUPQ^{Q}XZP^{[\]}ZQ^{U}XP^{V}MUZI_a^{[\]}MZPQ[aZ_QXXZS]UZQAPUZSc^{U}QZ^{Q}AP$ 

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 $: OVXT \land [RQ_UZNX_ \land [bUPUISOMQ`[ \land OSZNII` c [Y OZ c UXKNOZOPU R'[Y aZPQ^ MUIUS_[Y QY [PaXQ_[R `TQ@aOTNXF ^ NI_X aOZO@ AZXIQ > OVXLUIS B^[S^NY /@FA > Bfi [ SNUI NI aZPQ^ `NIPUIS [ R`TQQ Y \XOdUUQ_ [R \ OZNINX_O'OOZUIS NIP PUNSZ [ U UI `TQ R'_ NIP_QQ ZP` ^ Y Q_ Q_ [R \ OSZNII OEZ EOQ T``\_!! OXOVZUISZZ aOTNX ^ NI_ZOPaZVI!$ 

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$$\label{eq:started_formula} \begin{split} & \mathsf{FT}[\_Qc\ T[\ aZPQ^{\mathsf{N}}\ MV2 \setminus OZMNXCQ_{\mathsf{T}}\ US \ c\ TQ\ TQ^{\mathsf{N}}\ MV[\ fe[\ faX^{\mathsf{N}}\ U]\ aZPQ^{\mathsf{N}}\ MV2 \ [\ aZP\ aZU_{\mathsf{T}}\ _T[\ aXP\ aZPQ^{\mathsf{N}}\ MV2 \ [\ bQ^{\mathsf{N}}\ MV2 \ U]\ VIP\ Y\ [\ ZU[\ fuZS\ [\ R^{\mathsf{T}}\ QJ \ AVIV_{\mathsf{T}}\ OQQZ\ UZS\ (\ fS^{\mathsf{N}}\ M\ U)\ MU2 \ MU2$$

 $\begin{aligned} & 3 \text{XX} \text{NTT}[X] & \text{Se XXN}[ ^M[ ^UQ_UZ 3a_ ^ ^ MX0/MQQUbUZ SPaZPUZ S bUM? QPUDMQY a_ NQM00 QPUQP Ne ^TQ \\ & @MUUZ MX3_[ QUMUUZ [ RFQ_UZ S 3a ^T[ ^ UUQ_ /@3F3fi D5B3 > MNI ^M[ ^e 300 QPUMUUZ B^{1} S^{1} XI z z \\ & FTQE` NZPMP_MQ_Q Ne ^ TQ@MUUZ XXBNT[ X] & se 300 QPUMUUZ 3PbU[ ^e 5[ aZQX/@B335fi FTQ \\ & E` NZPMP_MQ_NMQP [ Z ^ TQUZ Q'ZMUUZ NX_ ^ NZPMP ; EA #' #+- E` NZPMP F[ ^? QPUDXX > MNI ^ MI[ ^UQ_Z z z QC LQVXNZP XXNI ^ M[ ^ MQ_MQ NQ0 QPUQP bUM 3@L a_UZ S ^ TQ_NY Q; EA #' #+- M_ ^ TQU^ \\ & NMUUZ T^ \_ !! c c c z' Q NZQPazVa! BMUQZ `! > MNZ300 QPUMUUZ fiz \end{aligned}$ 

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 $\begin{array}{l} \label{eq:states} \mathsf{E}[Z[S^{1}\mathsf{M}\mathsf{T}\mathsf{O}^{}](Y]\mathsf{U}SY \mathsf{OPU}\mathsf{O}\mathsf{M}\mathsf{A}X^{1}\mathsf{M}[aZP\,\mathsf{O}\mathsf{O}\mathsf{M}\mathsf{U}\mathsf{U}\mathsf{M}\mathsf{U}Z_Ya_{}^{}] \mathsf{N}_{2}a^{}] \mathsf{N}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[aZP\,\mathsf{O}\mathsf{M}\mathsf{M}\mathsf{U}\mathsf{M}\mathsf{U}Z_Ya_{}^{}] \mathsf{N}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[aZP\,\mathsf{O}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[Z_Ya_{}^{}] \mathsf{N}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}_{2}a^{}] \mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}\mathsf{M}[X] \mathsf{M}[X] \mathsf{M$ 

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1.

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	/	"*'\$0&:0('7&'/%+48'#,#/&5*'3%0'/2/ .645\$'#11+&8+5*%#650/

### ) COFOELY, . . VIXII JIDNEUN FS

 $\label{eq:solution} FT \_ UZF[ ^Y MUZ \_ UZ^2CPCP^{[ ^[ buPQSCZCYNXNPbU0D^{[ ^MODUUZC^{+} NZP_T[ a XP Z[ ^ NQ^2XUCP [ Z M_M _ a N_^Ua^2QF[ ^ ^[ ( ^ Q^M_Q Z C UT ^Q_Q QO ^{[ ^TQ M^2U2A} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_Q QO ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_N QD ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_N QD ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_N QD ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_N QD ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^TQ Z C UT ^Q_N QD ^{[ ^TQ M^2UA} M 00^{-} MZ OQ_[ RONOT OM_QNZP^{-} MZ OQ_[ RONO$ 

 $\begin{array}{c} \mathsf{TUX}^* \mathsf{TQ5}[\texttt{XCSQQZPQN6}[a^{-}[\texttt{QZ}_a^Q^*\mathsf{TM}\texttt{UZR}^*\texttt{Y}\texttt{MUZ}\texttt{U}\texttt{MODa}^*\mathsf{M}\texttt{Q}\texttt{MZ}^*\texttt{M}^*\mathsf{TQ}^*\texttt{U}\texttt{Q}[\mathsf{R} \\ \land \mathsf{Q}\texttt{M}\mathsf{M}\mathsf{M}\mathsf{U}\mathsf{Z}\mathsf{U}^*\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{Z}\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{U}\mathsf{Z}\mathsf{U}^*\mathsf{M}\mathsf{U}\mathsf{U}\mathsf{Z}\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{U}\mathsf{U}\mathsf{Z}\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{U}\mathsf{U}\mathsf{U}^*\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{U}\mathsf{U}\mathsf{U}^*\mathsf{U}^*\mathsf{M}\mathsf{Q}\mathsf{U}\mathsf{U}\mathsf{U}^*\mathsf{U$ 

) QQFOEJY - +POTJEFSBUIPOT (PS 10EJHFOPVT BOE +VN2V/SBN221 BOE 4JOHVJTUIDBN221, JNFSTF 8PQVN2UIPOT

- &z\$#
   FTQ^Q\_T[axPNQML\^[\\_/UV/QQYYaZU00/UZcUTNXKc[YQZBM\*U2aXM\*00/Q\_T[axPNQ\*N/0Z\*[

   QZ\_a^Q`TM1QYYaZU00/UZU\_000/MXIPaZPQ^\_[[PNec[YQZcT[MQR'[YQaXa\*NXKeNXIP

   XZSaU\_U00/XKePUbQ\_Q\[\aXX/UZ\_/XZ0APUZSc[YQZR'[YNX];ZPUSQZ[a\_NNX0/XS^[aZPfz]
- &z\$z\$ ;Z 3a\_`^\XXXII`TQ6Q\MYY QZ` [R;Y Y U\$^\VIUZ NZP 5UUGQZ\_TU\ [RRQ^\_8^QQ;Z`Q\ ^Q UZS EQ^bUQQ\_ `T^[ aST F;E @\VIUZ NXRT ^\ \Ub\VIQY OPUDVX\ ^\X0 UUZQ^\_PORZ OP M\_9QZQ^\XXB^\X0 UUZQ^\_NZP ? OPUDVXE\QQUXU\_\_Fi\ ^[ bUPUZS? OPUDMQ ^Q\VIQZ\VIXQ\_QbUQQ\_NZP`TQ.^^QQ\.`UZ Z\_`NRR`[ M^\XISQ NI\[ UZ`Y QZ`\_NZP \ ^[ bUPQ ^Q\_aX\_[ RY OPUDVXCQ\_Z8^QQUZ`Q^\ ^Q Q\_\_MQNXL[ Nb\VXXIVQUZ @Qc LQ\XXZPž

) COFOEJY. + POEJUPOTC WI I JH FSOSFVBNFOODF JO OF PONFIX JI OBSUDVNBSFU OLDUZ

) QQFOEJY / , FGOUJPOT BOE ) CCSFWBUPOT

 $\label{eq:started} FTQR[X][cUZS`NNVQPQ_NUX_`QY_NZP_NNVCbUVNUZ_a_QP`T^[aST[a``TU_`NQY_QZ`žFTQPQRZUUUZ_TNVbQNQQZ`NVVQZ R[Y`TQ@NUUZNX>UV/Me[R? QPUQUZQ_? QPUQUXEaNVQQ: QVPUZS_/? QE: fiPMINNVQQ cTQ'QNVbNXVNVQZ$ 

<FSN

, FGOUPO

) CCSFWBUIPO

+PN CLOFE . JSTU	5[Y NUZOP RV_``^VY Q_`O^_OYOOZ UZS`Q_` UZb[ X6Q_NZ	<b>C8</b> FE
<sin d3ffo.oh<="" ftufs;="" td=""><td>aX^M_[aZP_<b>0MINX</b>PMNX[[P`Q_`M##Ž#%L(c000V_</td><td></td></sin>	aX^M_[aZP_ <b>0MINX</b> PMNX[[P`Q_`M##Ž#%L(c000V_	
	\^OSZNZIO₽Ž	

	MQTUSTXé bimumixož	
6FHBUMØ8SFEJDUMØ >BI <b>VI</b> F	$\label{eq:ftqzqsMlbQ} \end{tabula} FTQZQSMlbQ \end{tabula} \end{tabula} \end{tabula} FTQZQSMlbQ \end{tabula} tabula$	@BH
6 VDI BNUSBOTTVIDFODZ	$3 \land 2 MNX a X M [Z[S M Te Y Q Ma Q Y QZ [R TQ [R U a Q NQT VZ P TQ R W X Q MX $	@F
7 FTUSIPM	$ \begin{array}{l} AZQ[R^{T}Q^{T}T^{Q}Q^{T}N^{T}Q \\ N[Pe^{Z}; ^{T} \sqcup MT[ ^{T} [ZQ^{T} N^{P}Q Pa^{T}Q^{T}S \land ^{Q}SZ N^{T}Q \\ A^{Q}P^{T} [ ^{T} Q^{T} M^{T} Q N^{T}Q N^{T}Q \\ A^{Q}Q P^{T} [ ^{T} Q^{T} Q N^{T} Q N^{T} Q N^{T} Q \\ M^{T} Q N^{T} Q \\ Q^{T} Q \\ Q^{T} Q \\ Q^{T} Q^{T} Q \\ Q^{T} Q^{T} Q \\ Q^{T} Q^{T} Q^{T} Q \\ Q^{T} Q^{T} Q^{T} \\ Q^{T} Q^{T} Q^{T} Q^{T} \\ Q^{T} Q^{T} \\ Q^{T} Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q^{T \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T \\ Q^{T} \\ Q^{T} \\ Q^{T \\ Q$	G7%
89FHOBODZ BTTPDJBUFE ONBITN B OSPUFJO)	3 \^[PaO [R`TQ\XXX0Z`MINZIP PQQIPaNI_QO^Q QP VZ`[`TQ Y MQ^ZNXQJQaXXIVJZ Pa^VZS\^QSZNZQež	B3BBŽ3
89FIJN ONZOUBUIPO HFOFUIDEJBHOPTJT	6Q QY UZIMU Z [R`TQZMa'Q [RM\MT[X] SUDXXQ ZPUU Z [^ PU_QM_QUZ `TQ[baY /feS[`Q/[^NXM_`[Qe_`\/U]^`[ UY \XXZ`MU Zž;`U_a_QP`[`Q_`QY M'e[_F[^_\_\QQROSQZQ UD [^QT^[Y [_[Y NXNNZ['Y NXUQ_NZP QZMXQ_`TQ_QXQU U Z [R aZNARQO QP QY N'e[_\^U]^`[UY \XXZ`MU Z NZP \^QSZNZQež	B96
8PTJUMØ 8SFEJDUMØ >BI <b>VI</b> F	FTQ\[_UUbQ\^QPVDUbQbN%bQU_`TQ\^[\[^\UZ[R\[_UUbQ ^Q_aX_VZ`Q`TM1MQ`^aQ\[_UUbQ^Q_aX_žFTQBBHU_Z[` VZ`^VZ_VD`[`TQ`Q_1_UPO\OZP_N%L[[Z`TQ	